Fundamentals of Turboexpanders
“Basic Theory and Design”

Edited Date: September 16, 2015
Presented By: Mr. James Simms
Simms Machinery International, Inc.
2357 “A” Street
Santa Maria, CA 93455
U.S.A.

About the Author
James (Jim) Simms has been involved with the design, manufacture, and service of Turboexpanders and other Cryogenic Rotating Machinery since 1969. He worked in the Engineering Department of various Turboexpander manufacturing companies until he founded Simms Machinery International, Inc. in 1988. The company primarily focuses on LNG Boil-off Gas Compressors aboard LNG Tankers (Ships). In 1994 he, along with others, founded Gas Technology Turbomachinery Services, Inc. to service Turboexpanders, primarily used in Gas Processing Plants.

Introduction/Description

The term “Turboexpander”, Figure 1, is normally used to define an Expander/Compressor machine as a single unit. It consists of two (2) primary components; the Radial Inflow Expansion Turbine and a Centrifugal (Booster) Compressor combined as an assembly. Its Wheels are connected on a single Shaft. The Expansion Turbine is the power unit and the Compressor is the driven unit.

Figure 1
Turboexpander (Expander/Compressor) Assembly Cross-Section Drawing

In a Gas Processing Plant, the purpose of the Turboexpander is to efficiently perform two (2) distinctly different, but complimentary, functions in a single machine. The primary function is to efficiently generate refrigeration in the process gas stream. This is done by the Expansion Turbine end efficiently extracting the potential heat energy from the gas stream, causing it to cool dramatically. This extracted energy is converted to mechanical energy to rotate the Shaft to the Booster Compressor end of the Turboexpander, which partially recompresses the residue gas stream. The Turboexpander operates according to the thermodynamic and aerodynamic laws of physics. When designed properly, the Turboexpander can yield very high efficiencies at the “Design Point” and reasonable efficiencies at other, or “Off-Design”, Points.

Application

The typical Turboexpander process installation is shown in Figure 2, Simplified Process Schematic. High pressure, moderately cold gas flows into the Expander section of the Turboexpander. The gas flows through the Expander Variable Inlet Nozzles (Guide Vanes) and then through the Wheel, exhausting at a lower pressure and at a substantially colder temperature. Gas flows from the Expander to the Demethanizer, where condensate is removed.

It should be noted that the Expander Nozzles are used to control the gas flow rate in order to maintain the pressure in the Demethanizer. The Residue Gas from the Demethanizer Tower flows through the Feed Gas Heat Exchanger and then to the Booster Compressor end of the Turboexpander. The efficiency of the Booster Compressor is very important, as it can improve the expansion process for more refrigeration as well as more efficiently use the power extracted by the Expander. While the engineering process to properly design a high
efficiency Turboexpander is very complex, requiring computerized analytical tools, the basic initial sizing process can be simplified by using certain basic equations and assumptions.

**Preliminary Size Calculations**

The Turboexpander operation is best described as a dynamic system which responds to the Process Stream variations. To start the initial sizing design process, a fixed set of Process Stream parameters, or "Design Point", must be established. This "Design Point" is normally set by the plant process engineer's system analysis in the case of new plants. In the case of a Turboexpander re-design in an existing plant, the actual operating condition will dictate the new "Design Point".

The parameters required to size the Turboexpander are:

- Gas Composition
- Flow Rate
- Inlet Pressure
- Inlet Temperature

Normally, the Expander Outlet Pressure is determined by the performance of the Booster Compressor's efficiency through a complex iterative analysis. For this simplified sizing exercise, we will assume the value of the Expander Outlet Pressure.

**Energy Extraction**

Where does the energy come from? With reference to Figure 3, which shows the typical Expansion Process Graph, we see the process in terms of change of energy, or $\Delta h$, with units of btu/lb, both in the Ideal Process (Isentropic, or 100% efficient) and in the Actual Process, or real terms.

These have been designated $\Delta h_i$ and $\Delta h_o$, respectively. The ratio of these is the definition of the Isentropic efficiency, $\eta_e$, of the Expander (i.e., $\eta_e = \Delta h_o / \Delta h_i$). For example, if $\Delta h_o = 34$ btu/lb, and $\Delta h'_i = 40$ btu/lb, then:

$$\eta_e = \frac{\Delta h_o}{\Delta h'_i} = \frac{34}{40} = .85 \text{ or } 85\%$$

With the $\Delta h'_i$ and $\eta_e$ values known, we then only need the gas mass flow rate, $w$, to calculate the Horsepower developed by the Expander. Since the mass flow rate is normally given by the process engineer, we now have enough information to calculate the Horsepower with the following formula:

$$HP_{\text{Expander}} = \left( \frac{778}{550} \right) \times \Delta h'_i \times w \times \eta_e$$

$778 / 550 = 1.4145$

(This is the constant value to change btu units into Horsepower terms)

We will assume the mass flow rate, $w = 20$ lbs/second. So, if $\Delta h'_i = 40$ btu/lb, $w = 20$ lbs/second, and $\eta_e = .85$, then:

$$HP_{\text{Expander}} = 1.4145 \times 40 \times 20 \times .85 = 961.9$$
Horsepower Balance

The HP developed by the Expander must be absorbed in order to prevent over-speeding. The Bearings and the Compressor absorb this power to create a balance. The Horsepower Balance formula is:

\[ \text{HP}_{\text{Expander}} = \text{HP}_{\text{Compressor}} + \text{HP}_{\text{Bearings}} \]

Example
Let’s assume that the Bearings consume a total of 30 Horsepower, which is subtracted from the Expander Horsepower. If we rearrange the formula above, then the available Horsepower to the Compressor is:

\[ \text{HP}_{\text{Compressor}} = \text{HP}_{\text{Expander}} - \text{HP}_{\text{Bearings}} = 961.9 - 30 = 931.9 \text{ Horsepower} \]

How Does the Booster Compressor Use Horsepower to Create Pressure?

The pressure rise through the Compressor is a function of the adiabatic \( \Delta h' \), or Head Rise, developed by the Compressor and its efficiency, \( \eta_c \), to satisfy the Horsepower balance formula. The Horsepower formula for the Compressor is similar to the Expander, but slightly different as it is consuming power. The formula is:

\[ \text{HP}_{\text{Compressor}} = \frac{1.4145 \times \Delta h'_{\text{Adiabatic}} \times w}{\eta_c} \]

The value of 1.4145 is, again, the constant to convert the units into Horsepower, just the same as the Expander formula. \( \Delta h'_{\text{Adiabatic}} \) is the ideal Head Rise, or energy change, from the Inlet Pressure to the Discharge Pressure of the Compressor. The units are btu/lb of gas flow. \( W \) is the symbol for mass flow through the Compressor, and the units are lbs/second.

The mass flow rate, \( w \), for the Compressor, is usually given by the process engineer. With the available Compressor Horsepower known, we can rearrange the Compressor Horsepower formula to calculate the \( \Delta h'_{\text{Adiabatic}} \) Head Rise, as follows:

\[ \Delta h'_{\text{Adiabatic}} = \frac{\text{HP}_{\text{Compressor}} \times \eta_c}{1.4145 \times w} \]

We start by assuming a reasonable efficiency, \( \eta_c \), on the Compressor, such as 75%. Figure 4 shows a typical Compression Process in which the ideal head rise, \( \Delta h' \), is lower than the actual Head Rise, \( \Delta h_o \), required to achieve the Discharge Pressure, the ratio of \( \Delta h' / \Delta h_o \) = efficiency of the Compressor, or \( \eta_c \).

\[ \begin{align*}
\text{Figure 4} & \\
\text{Compression Process} & \\
\end{align*} \]

Discharge Pressure Calculation

Now, having calculated the \( \Delta h'_{\text{Adiabatic}} \), we can calculate the pressure rise through the Compressor by the following formula:

\[ P_{\text{out}} = P_{\text{in}} \times \left[ 1 + \frac{\Delta h'}{C_p T_1 Z} \right]^{\gamma / (\gamma - 1)} \]

Definition of Terms:
- \( P_{\text{out}} \) = Discharge Pressure, PSIA
- \( P_{\text{in}} \) = Inlet Pressure, PSIA
- \( C_p \) = Specific heat at constant pressure
- \( T_1 \) = Inlet temperature, °R (°R = °F + 459.69)
- \( Z \) = Compressibility Factor
- \( \gamma \) = Greek Symbol "Gamma", stands for the Ratio of specific heats
To be accurate, the values of \( C_p, Z, \) and \( \gamma \) are best derived by a good equation of state computer program; however, for this exercise, we will assume certain values for these (\( T_1 = 60^\circ F, C_p = .54, Z = .98, \) and \( \gamma = 1.3 \)) and solve the above formula.

So, to repeat the formula for the Compressor if the Inlet Pressure equals 145 PSIA, the Discharge Pressure will be calculated:

\[
P_{\text{out}} = P_{\text{in}} \times \left[ 1 + \frac{\Delta h'}{C_p T_1 Z} \right]^{\gamma / \gamma - 1}
\]

\[
= 145 \times \left[ 1 + \frac{18.30}{.54 \times (60 + 45.969) \times .98} \right]^{1.3 / .3}
\]

\[
= 145 \times (1.0665)^{4.333}
\]

\[
= 145 \times 1.322
\]

\[
= 191.7 \text{ PSIA}
\]

How is the Turboexpander Design Speed Determined?

This is done by using the term **Specific Speed**, or \( N_s \), of the Expander Wheel:

\[
N_s = \frac{N \times \sqrt{\text{ACFS}_2}}{(778 \times \Delta h'_{s})^{.75}}
\]

From earlier pages, \( \Delta h'_{s} = 40 \text{ btu/lb} \)
Assume a Specific Speed, \( N_s = 75 \) for good efficiency
Assume, \( \text{ACFS}_2 = 25.0 \) (\( \text{ACFS}_2 \) is the Actual Cubic Feet per Second of volumetric flow at the outlet of the Expander).
Rearrange the formula and calculate the speed, \( N \):

\[
N = \frac{N_s \times (778 \times \Delta h'_{s})^{.75}}{\sqrt{\text{ACFS}_2}}
\]

\[
= 75 \times (778 \times 40)^{.75}
\]

\[
= 75 \times 2343
\]

\[
= 35,146 \text{ RPM}
\]

How to Approximate the Expander Wheel Diameter

First, for good efficiency, the term \( U/C_o \) should equal about 0.7. The \( C_o \) term is known as the spouting velocity of the gas from the Nozzles into the Expander Wheel. The formula is:

\[
C_o = \sqrt{\frac{2 \times g \times J \times \sqrt{\Delta h'_{s}}}{\Delta h'_{s}}}
\]

\[
= \sqrt{\frac{2 \times 32.2 \times 778 \times \sqrt{40}}{40}}
\]

\[
= 223.8 \times \sqrt{40}
\]

\[
= 1415 \text{ ft/sec}
\]

Since the term \( U, \) Wheel Tip Speed, needs to be 0.7 of \( C_o \), then:

\[
U = 0.7 \times C_o
\]

\[
= 0.7 \times 1415
\]

\[
= 990 \text{ ft/sec}
\]

Definition of Terms:
- \( U \) = Tip Speed of Wheel, ft/sec
- \( C_o \) = Spouting Velocity, ft/sec
- \( g \) = 32.2 ft/sec^2
- \( J \) = 778 ft-lb/btu

The term, \( U \), is the peripheral velocity of the Expander Wheel (tip speed) which is calculated by:

\[
U = \frac{\text{Diameter (inches)} \times \text{RPM}}{229.2}
\]

So, to rearrange the formula to solve for the Wheel Diameter:

\[
\text{Diameter} = \frac{U \times 229.2}{\text{RPM}}
\]

\[
= \frac{990 \times 229.2}{35,146}
\]

\[
= 6.46 \text{ inches}
\]

How to Approximate the Compressor Wheel Diameter

The following formula can be used to approximate the Compressor Wheel Diameter:

\[
\Delta h'_{\text{Adiabatic}} = \frac{U^2 \times \psi}{g \times J}
\]

From previous pages, we calculated the Compressor \( \Delta h'_{\text{Adiabatic}} \) to equal 18.30 btu/lb. Let the head
coefficient, $\Psi$, be equal to .4 and rearrange the formula to solve for $U$:

$$U^2 = \frac{\Delta h' \times g \times f}{A}$$

$$U = \sqrt{\frac{(\Delta h' \times g \times f)}{A}}$$

$$U = \sqrt{\frac{18.30 \times 32.2 \times 778}{A}}$$

$$U = 1070.6 \text{ ft/sec}$$

Now calculate the Compressor Wheel Diameter from the equation:

$$U = \frac{\text{Wheel Diameter} \times \text{RPM}}{229.2}$$

From the Expander analysis, the design speed was calculated to be 35,146 RPM. So rearranging and solving the above formula:

$$\text{Compressor Wheel diameter} = d_C$$

$$d_C = \frac{U \times 229.2}{\text{RPM}}$$

$$d_C = \frac{1070.6 \times 229.2}{35.146}$$

$$d_C = 6.98 \text{ inches}$$

**Caution Note**: While the above method does give the steps necessary to do the initial rough sizing of the Expander and Compressor Wheels, it is vastly over simplified. In the above, certain values were assumed for specific speed, efficiency, head coefficient, etc. These assumptions were used for ease of demonstrating the formulas only. These assumptions MUST NOT be used as "rule of thumb" values, as they may give false results in an actual case. Any actual design analysis should be performed by an experienced Turboexpander Design Engineer.
Lube Oil System

The Lube Oil Pressure supplied to the Bearings is controlled at a pressure higher than the Reservoir, typically this pressure is 150 PSID controlled by a Differential Pressure Regulating Valve relieving excess oil to the Reservoir. Dual, Electric Motor Driven, Lube Oil Pumps (one Main and one Standby) take suction from the Reservoir providing oil through the Cooler or bypassing the Cooler via the Temperature Control Valve as required to maintain a preset oil supply temperature to the Bearings. The oil is then filtered by one of the Dual Filters. At this point the oil pressure regulates to the proper Differential Pressure. An Accumulator is installed to store oil for emergency coast down in case of electrical power outage. The oil then travels to the Bearings. Some systems have an oil flow rate measuring device in the oil supply line to the Bearings.

Shaft Sealing System

The Shaft Seals are Labyrinth type using Seal (Buffer) Gas to prevent cold gas migration into the Bearing Housing and prevent oil leakage into the process stream.

The Seal Gas system typically uses warm process gas that has been filtered. The pressure to the Labyrinth Seals is maintained at a Differential Pressure of typical 50 PSID above the pressure behind the Expander Wheel. This is accomplished by use of a Differential Pressure Regulator sensing and floating with the Expander Back Wheel Pressure.

The small amount of Seal Gas going across the Seal towards the Bearing Housing mixes with the oil and drains to the Reservoir. In the Reservoir, this separates from the oil and then is vented through the De-misting Pad out into the Booster Compressor Suction. Basically, the Turboexpander System Pressure automatically floats with the process pressure via the Compressor Suction.

For control of the Turboexpander at start-up, a local Hand Indicating Controller (HIC) is provided to adjust the Expander variable Nozzles to control the Gas Flow (and Expander Speed) during start-up.

Primary safety instrumentation includes Speed Probe, Vibration Probe, Bearing Temperature RTDs, Thrust Oil, and Seal Gas Differential Pressure Switches. A first-out Annunciator System provides an indication of the initial cause of a shut-down.

The Automatic Thrust Equalizer System vents pressure from behind the Compressor Wheel to try to equalize the oil pressure measured at each Thrust Bearing.

Description of Thrust Balance System

The purpose of this system is to control the Axial Thrust of the Expander/Compressor Rotor within safe load limits, in either direction, on the Thrust Bearings.

The original version (Figure 7) operates in the following manner. The oil pressure at the Thrust Face of each Bearing is measured and connected to two opposite ends of a Piston Chamber. A Piston Shaft is connected with an Internal Seal to a Gate Valve that is connected between the pressure port behind the Compressor Wheel and the suction of the Compressor.

![Original Automatic Thrust Equalization Design](image)

The GTS improved version (Figure 8) operates in a similar manner as the original system, but with the addition of a constant feed oil supply for improved response to Thrust variations. The Gate Valve is
replaced by a "balanced piston" Spool Valve, which is less prone to sticking (less resistance) than the Gate Valve, and therefore more responsive. The Spool Valve is connected between the pressure vent port behind the Compressor Wheel and the suction of the Compressor, just as the original system.

The Thrust balancing is accomplished by maintaining or decreasing the pressure behind the Compressor Wheel. This is accomplished when the Thrust force (oil pressure) on the Compressor Thrust Bearing is increased, which causes the Piston to slowly open the Spool Valve, thereby reducing the pressure in the area behind the Compressor Wheel causing the load on the Compressor Thrust Bearing to reduce. The opposite occurs when the load is increased toward the Expander Thrust Bearing. In turn, the Spool Valve will tend to close. With the GTS version, the end of the Thrust Equalizing Valve can be viewed and verified as to which direction the Thrust Valve is in.

**Description of Surge Control System**

**The Surge Condition**
The phenomenon of "surge" in an axial or centrifugal Compressor occurs when the flow is reduced sufficiently to cause a momentary reversal of flow. This reversal tends to lower the pressure in the discharge line. Normal compression resumes, and the cycle is repeated. This cycling, or surging, can vary in intensity from an audible rattle to a violent shock. Intense surges are capable of causing complete destruction of the components in the Compressor such as Blades, Bearings, and Seals. An Anti-Surge Control system is therefore recommended to provide positive protection against surging or cycling.

**Flow Relation in the Compressor**
The performance map of the Compressor, typically supplied by the Compressor manufacturer, is a plot of Pressure increase (head) vs. Capacity (flow) over the full range of operating conditions. At any given Compressor speed, a point of maximum discharge pressure is reached as the flow is reduced. This is indicated at points A, B, C, D, E, and F in Figure 9 below. The line connecting these points describes the surge limit line, which separates the region of safe operation from the surge area.

As flow is reduced, the pressure in the Compressor tends to be lower than the discharge pressure and a momentary reversal of flow occurs. This reversal then tends to lower the pressure at the discharge, allowing for normal compression until the cycle repeats itself.
Surge Control

The most common method of Surge Control uses the Compressor $\Delta P$ to represent “head” and the differential pressure across an Inlet orifice (called “$h$”) to represent capacity. The function of the Surge Control system is to keep the ratio of $\Delta P/h$ from exceeding the slope of the surge line.

To provide some factor of safety, a control line (Set Point) should be established to the right of the surge line, as shown in Figure 10 below.

A typical anti-Surge control system block diagram is shown on the attached sketch, Figure 11. In operation, the Inlet Flow is measured differentially through an Orifice plate using a DP Transmitter, with its output connected to a Ratio Station (multiplier) and then transmitted as a set point to the Controller (PID). The Inlet and Discharge pressures of the Compressor are also measured using a DP Transmitter and transmitted as the Process variable to the Controller.

The output of the Controller operates a bypass valve that recycles the gas back to the Inlet of the Compressor. In general, when the two (2) signals become equal within the bounds of the Proportional Band range, the Controller will output a signal to open the Bypass (Recirculation) Valve. If the Flow rate is too low (i.e., near surge) then the Bypass Valve should open and allow additional flow to re-circulate back to the Compressor as necessary. In normal operation, the Bypass Valve is closed to prevent pressure losses and it opens only to prevent surge.